2950 Niles Road, St. Joseph, MI 49085-9659, USA
269.429.0300 fax 269.429.3852 hq@asabe.org www.asabe.org

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Modeling Energy Consumption for Crushing of Roundwood as a First Stage of Feedstock Preparation

James H. Dooley

Forest Concepts, LLC

Christopher J. Lanning

Forest Concepts, LLC

David N. Lanning

Forest Concepts, LLC

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Abstract. *Our objective is to apply an understanding of the modes of failure and structural biology to substantially reduce the comminution energy required to produce bioenergy feedstocks. This paper explores the modes of failure for wood materials subject to crushing forces and how they could be used to develop a mathematical model of crushing forces for a round roller acting on a round log. Our hypothesis is that crushing or roller-splitting is a low-energy and effective method to reduce the thickness of round logs and change the resulting shape for subsequent processing. Modes of failure during crushing suggest that a mathematical model could be developed to estimate required crushing forces and energy for round logs. Such a model has been called for since early work by the USDA Forest Products Laboratory and Tennessee Valley Authority more than 30 years ago. A model was developed by the authors and experimentally validated for the case of a round roller compressing a round log.*

Keywords. Woody biomass, bioenergy, comminution energy, log, forestry, forest products

Introduction

Comminution energy accounts for a significant portion of the total energy consumed during the production of biofuels from lignocellulosic feedstocks. This is particularly true for the case of forest materials that start out as logs at the point of harvest. The conventional pathway is for harvested logs to be chipped or shredded at a roadside to facilitate trucking and downstream handling. In-woods chipping and shredding consume large quantities of fossil fuel to power the equipment.

An objective of the bioenergy development program at Forest Concepts is to take advantage of natural modes of failure that can be induced within cellulosic raw materials to reduce the energy consumption for comminution and/or densification. Crushing (known as hay conditioning in the forage industry) has been demonstrated by others to be of relatively low energy intensity and to produce flexible and open structured materials that may dry faster under natural conditions, be densified into round or square bales to enable handling, and to be readily processed into bioproducts in the case of scrim wood.

The authors conducted a number of laboratory experiments that validated earlier research on the forces and energy required for crushing wood. They also found that all prior work had been empirical in nature. While earlier authors suggested that mathematical models of crushing failure in wood would be useful, none completed the task to a point of publication.

This paper offers a first attempt to develop a mathematical model of the crushing forces and energy needed for crushing round logs using pairs of round rollers. The laboratory experiments and model development were conducted within the context of a low-energy comminution research project funded by the U.S. Department of Energy (DE-SC0002291). The authors present this model and approach to others for improvement.

Safety Emphasis

Crushing and subsequent baling or bulk handling of forest logs and residuals is much safer than chipping and shredding. The safety zone around a crusher and baler system is only a few meters – not much more than the length of the woody materials being handled. In contrast, the safety zone around a chipper is typically 25 to 50 meters, and for tub grinders is 75-100 meters. Chippers and grinders tend to throw large chunks of material up into the air and for long distances while crushers and balers do not.

Literature Review

The earliest comprehensive discussion of low-energy comminution for wood based on modes of failure appears to be the work of Keith C. Jones under contract to Forest Engineering Research Institute of Canada (Jones 1981, 1981). Jones assumed that the energy to shear across grain in black spruce wood was 100 times the published energy for parallel-to-grain shear. His values for black spruce were 0.1 J/cm^2 for parallel-to-grain shear and 1.0 J/cm^2 for shear perpendicular to grain. He further estimated that energy requirements to produce cubic particles of any size could be estimated for wood with an assumed specific gravity of 0.38 using the equation:

Surface energy requirement = 5.4/x MJ/ODt

Where: x = length of each side of the cube in cm.

Jones summarized the work of Papworth and Erickson on chipping aspen, hemlock and hard maple logs where the chipper acted perpendicular to grain to produce 12 mm long chips (Papworth and Erickson 1966). Papworth found that the Machine Energy (energy consumed by the chipper head) was 16.9, 19.9 and 22.6 MJ/ODt respectively for aspen, hemlock, and hard maple.

Jones compared chipping to hammer mills which are much more tolerant of wood piece size, debris, rock, etc. He reported that the Machine Energy for hammer mills ranged from 80 MJ/ODt for 70 mm average particle size to more than 450 MJ/ODt for 10 mm average particle size.

In his quest for lower energy comminution methods, Jones explored roll crushers that work by causing splitting parallel to grain. He suggested that splitting rolls may increase the uniformity of strand width without increasing the crushing energy. A more advanced concept for roll crushers was reported by Jones as the “Russian Roll Crusher” which had both radial and longitudinal knives (like a pasta cutter). The machine reportedly processed 300 mm diameter roundwood into 200 mm chunks through the use of 75 kw of electric power and at a production rate of 7.5 ODt per hour (93 m^3/hr). This rate computes to a machine energy requirement of approximately 18 MJ/ODt.

Some time prior to 1980, the Tennessee Valley Authority (TVA) began development of the “TVA Fiberizer” to apply low energy crushing and tearing methods to produce composite board furnish (Harvey 1972). The experimental fiberizer was estimated to consume 79 MJ/ODt at a productivity rate of 1.5 ODt/hr.

Jones evaluated the energy requirements for the Stuart Virginia Polytechnic Institute (VPI) baler (Walbridge and Stuart 1981) as an alternative to chipping when the primary objective was to improve transport efficiency. Jones estimated the Machine Energy for the VPI baler to be approximately 1.1 MJ/ODT for softwood slash to compact the material to approximately 50% of its original volume. He further estimated that the bale shear consumed an additional 0.97 MJ/ODt to shear logs and slash to 1.1 m length. By using the shearing data from Arola (Arola 1971), Jones revised his estimate of the baler’s Machine Energy to be 2.5 MJ/ODt. Jones favorably compared baling at 2.5 MJ/ODt to the otherwise low energy chipping at 10 MJ/ODT for similar materials. He further calculated that if forest slash was baled in the field and then chipped at an industrial site the combined baling and chipping Machine Energy would be approximately 17 MJ/ODt, with less than half of that from fossil fuel energy.

In 1984, the Tennessee Valley Authority (TVA) and the Forest Engineering Research Institute of Canada (FERIC) teamed to explore roll splitting as a means to comminute wood as a biofuel with less energy consumption than cutting or chipping (Du Sault 1984, 1985). An additional objective was to mechanically dewater wood through crushing. They found that full crushing doubled the energy consumption with no additional dewatering as compared to crushing only enough to split the wood into small sections and splinters. They experimented with a single pair of rolls and a sequence of two pairs of rolls. In both cases the upper rolls were smooth or made from “diamond plate” and the lower rolls contained cross bars with 50mm tall triangular teeth. They found that front end of each log tended to be poorly crushed due to poor feeding into the nip of the rolls. This was attributed to a need to release down-pressure in order to begin the feeding of each log section into nip. The upper roller was held down by 101 mm (4 in) diameter hydraulic cylinders located above each end support.

Table 1. Crushing forces calculated from data and reported energy consumption for double-roll splitter from Du Sault (1985).

Species	Reported hydraulic crushing pressure for blocks 100 – 120 mm diameter.	Calculated crushing force with 2 ea. 4-inch cylinders	Cycle comminution energy	Machine Comminution Energy
Yellow Poplar	4825-6205 kPa (700-900 psi)	78-101 N (17,500-22,600 lbf)	622 MJ/ODt	115 MJ/ODt
Red Maple	6895-8275 kPa (1000-1200 psi)	112-134 N (25,100-30,200 lbf)	732 MJ/ODt	187 MJ/ODt
Loblolly Pine	5515-6895 kPa (800-1000 psi)	89-112 N (20,100-25,100 lbf)	630 MJ/ODt	139 MJ/ODt

Du Sault defined the term “machine comminution energy (MCE)” as being the incremental energy consumed by the working tools during actual work. The MCE is expressed as Megajoules per oven dry tonne (MJ/ODt).

$$E_M = ((E_{total} - E_{no\ load}) \times T) / W_d \quad \text{Equation 1}$$

Where: E_M = Machine energy

E_{total} = Total measured energy consumption

$E_{no\ load}$ = Energy consumption at no-load

T = Total time to process biomass

W_d = Dry weight processed during time T

Du Sault additionally defined “cycle comminution energy (CCE) to be the total energy per unit processed including no-load energy.

Du Sault compared different versions of the roll splitter that were tested only on hardwood species to report MCE values of 32 MJ/ODt for the earlier TVA single roll splitter and 66 MJ/ODt for a dual roll splitter. He did not explain why the values are significantly lower than those reported in the experimental data summarized above in Table 1. In any case, Du Sault reports that production of maple wood pulp chips typically consumes 22 MJ/ODt (Papworth and Erickson 1966) and hogging of hemlock wood consumes 100 MJ/ODt. He concludes that chipping is more energy efficient than crushing, while hogging is less energy efficient. However, since his objective was to promote natural air drying, the net energy content of air dried wood from the crushing process was very high since neither chips nor hog fuel dry much after processing and piling.

The USDA Forest Service Southern Forest Research Station assumed responsibility for continuing the work of TVA and FERIC to further develop biomass crushing and splitting for the purpose of accelerating natural air drying (Ashmore, Sirois, and Stokes 1986; Barnett, Sirois, and Ashmore 1986; Curtin, Sirois, and Sturos 1987; Sirois and Ashmore 1986; Sirois, Rawlings, and Stokes 1991). Instead of crushing to the point of pressing water out of the wood, the USFS engineering team sought to apply the principles of hay conditioning rolls to woody biomass and small diameter trees up to approximately 180 mm (8 inches) diameter. They found that smooth rolls or rolls with cross bars were effective to crush wood stems at all moisture contents. Their preferred operating speed was 15 meters per minute (50 fpm). The action of crushing rolls served to split the biomass stems and that further crushing had no additional benefit to the rate of natural air drying. The recommended design included a hydraulically biased initial set of crushing rolls followed by a second set of rolls that had a fixed gap of either 12.5 mm or 25 mm (1/2 or 1 inch). Both sets of rolls were 500 mm (20 inches) diameter.

The patent literature includes log crushers that date to the 1880's. A few of the more illustrative patents are described below.

William Cornell (USP 773479, 1904) described a wood chip and chunk crusher for making mechanical pulp. His crusher included smooth rolls and paired rolls that included longitudinal corrugated ribs on the lower roll with curved herringbone ribs to spread the fibers on the upper rolls. Additionally, the invention included mechanisms to reciprocate the rolls endwise to create scrubbing action, and ability to run the pairs of rolls at different speeds to further scrub the chips. Cornell's preferred solution included three sets of rolls. The first were smooth to crush the chips. The second included closely spaced corrugations and fine herringbone groves. The third included wider spaced corrugations and herringbone ribs.

The earlier mentioned work at Tennessee Valley Authority resulted in Herbert Harvey (USP 3674219, 1972) that describes a method for converting solid pieces of timber into splinter-like strands of wood with fairly uniform thickness for the purpose of manufacturing fiber strand-board products. Harvey's "wood defibrading apparatus" a series of seven pairs of rolls with helical, corrugated and other patterns. Harvey was particularly interested in increasing the utilization of small diameter, debarked logs and greatly increase the volume recovery as compared to sawing. He found that the knot material tended to crumble and be removed from the scrim mat. This improved the strength of resulting strand board.

Harvey reported that the energy required was minimized if the moisture content of the logs was at least 30 percent (wb) and higher if the logs were green. Further, he reported that the compression strength of logs was reduced by 80 percent when the logs were preheated to near 100 ^{o}C. Harvey overdrove the top rolls in each pair to improve the separation of fibers as the logs were crushed. Through experimentation, he determined that the maximum downforce on the first crushing rolls should be 93,000 pounds to maintain contact pressures between 1,500 and 3,000 psi., while the later stages may require less than half that as the mat becomes spongy and spreads out. The last stage of the Harvey process was to feed the material laterally into a "srubber jaw" machine that work the mat bundle to pull it apart. The fiberized mat is then fed to a lay-up and molding station to form new strand board products.

A US patent was issued in 1980 to Coleman (USP 4232067, 1980) that described a method to crush and fiberize logs into webs that could be reformed into useful shapes. A second patent to Coleman (USP 4711684, 1987) described an improved process. The 1987 patent features the system to reciprocate the top roll so as to scrub the wood fibers as they are crushed. This appears to be a modernization of the methods previously described by Cornell in 1904. The Coleman "067" patent appears to be informed by the TVA/FERIC/USFS projects in that it describes the initial log crushing with rolls that are "textured, serrated or toothed" and driven at a preferred speed of approximately 40 fpm. The Coleman "684" patent proposes a roll form that has circular grooves that are 1-10 mm deep and with 4 mm wide lands between grooves. The grooved rolls enhance the fiber scrubbing action when the rolls are reciprocated. Coleman states that the amplitude of reciprocation must be experimentally determined, but typically is found to be within the range of 40-200 mm. Too fast of a reciprocation tends to tear the mat. He also observes that if the machine includes multiple sets of reciprocating crusher rollers, the reciprocation can get out of phase and break the mat. On the same day in 1987, Coleman was awarded a patent for the wood reconsolidation process (USP 4711689). The “689” patent does not include any new information about the engineering of mat forming processes.

Unfortunately, the technical and patent literature does little to tell us the pressures, forces, modes of failure and other first principles that underlie design of wood crushing and processing equipment. Measurements of pressures, forces and energy consumption were crude at best. The literature does suggest a number of design features for roll configurations, roll reciprocation, and log preparation that may save us time and experimentation.

Mode of Failure in Wood during Crushing

The initial mode of failure within round logs subjected to crushing forces normal to the longitudinal axis of the log is triggered by Poisson’s forces that cause the log to split as the internal stress exceeds the tangential-to-grain tensile strength of the wood.

Table 1. Poisson’s ratios for various species at approximately 12% (db) moisture content (Forest Products Laboratory 1999).

Species	μ_{LR}	μ_{LT}	μ_{RT}	μ_{TR}	μ_{RL}	μ_{TL}
Douglas-fir	0.292	0.449	0.390	0.374	0.036	0.029
Lodgepole Pine	0.316	0.347	0.469	0.381	-	-
Maple – Red	0.434	0.509	0.762	0.354	0.063	0.044

The first letter of the subscript refers to the direction of applied stress and the second letter to the direction of lateral deformation.

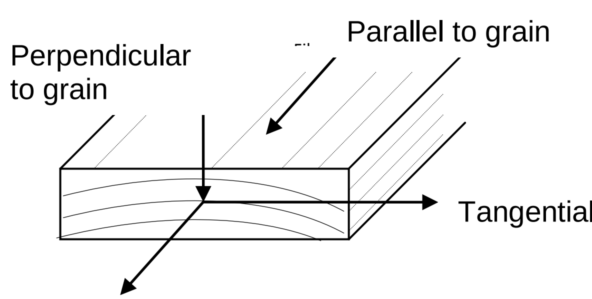


Figure 2. Three principal axes of wood with respect to grain direction and growth rings (Forest Products Laboratory, 1999)

In the case of round logs under radial compression, the mode of failure is a result of force in the radial direction and Poisson's forces in the tangential direction. Thus, the μ_{RT} values in the above Poisson's ratio table are of relevance.

Table 2. Strength properties of wood (Forest Products Laboratory 1999).

Species	Specific Gravity	Modulus of rupture (kPa)	Compression parallel to grain (kPa)	Compression perpendicular to grain (kPa)	Shear parallel to grain (kPa)	Tension perpen-dicular to grain (kPa)	Toughness Radial (J)	Toughness Tangential (J)
Douglas-fir (coast) Green	0.45	53,000	26,100	2,600	6,200	2,100	3,400	5,900
12% MC (dwb)	0.48	85,000	49,900	5,500	7,800	2,300	3,300	5,900
Lodgepole Pine Green	0.38	38,000	18,000	1,700	4,700	1,500	2,600	3,400
12% MC (dwb)	0.41	65,000	37,000	4,200	6,100	2,000	-	-
Maple – Red Green	0.49	53,000	22,600	2,800	7,900	-	-	-
12% MC (dwb)	0.54	92,000	45,100	6,900	12,800	-	6,000	5,900
Poplar – Black Cottonwood Green	0.31	34,000	15,200	1,100	4,200	1,900	-	-
12% MC (dwb)	0.35	59,000	31,000	2,100	7,200	2,300	-	-

In the table above, mechanical properties of select wood species are listed from the Wood Handbook. Wood is an orthotropic material in that its properties vary depending on the orientation of the material to any applied load.

- Modulus of rupture – the maximum load-carrying capacity of a wood member in bending and is proportional to the maximum moment borne by the specimen. This value is useful for "tenderizing" wood by passing it through a serpentine set of rollers such that it is bent up and down or including side-to-side bending such that the wood is broken in bending.
- Compressive strength parallel to grain – Maximum stress to yield point sustained by compression parallel to grain having a length to least dimension of less than 11.
- Compressive strength perpendicular to grain – Reported as stress at proportional limit where the deformation no longer behaves as a plastic material.
- Shear strength parallel to grain – Ability to resist internal slipping of wood structure along the grain.

- Tension strength perpendicular to grain – Resistance of wood to forces acting across the grain that tend to trigger splits. Typically reported as the average of radial and tangential strength.
- Toughness – Energy required to initiate rapid complete failure in a centrally loaded bending specimen.

Mode of Failure Experiments

We crushed cubic Douglas fir wood blocks to assess the mode of failure and obtain first-order data relating the mode of failure to compression forces. Two block sizes were used – 50mm cubes and 75mm cubes. From the Wood Handbook data, we would expect the blocks to begin to crush at a platen pressure of approximately 40,000 kPa assuming the blocks were at an equilibrium moisture content of 20%(wwb). What we found was that the blocks uniformly began to fail at a pressure of only 5,000 kPa. The mode of failure suggested that Poisson's forces were causing the block to split into sections from tangential tension rather than simply crushing in pure compression. We can thus look at the effect of Poisson's ratio to see that the Poisson's force tangential to grain as a result of pressure parallel to grain is approximately 0.4 times the compression force. Thus, at a platen pressure of 5,000 kPa, the Poisson's pressure in the lateral direction will be approximately 2,000 kPa. That value is approximately the same as the published tensile strength of the wood in that direction. This result is entirely consistent with the findings of Lanning (Lanning et al. 2008) in earlier Forest Concepts experiments that concluded that Poisson's forces are more important than simple compression forces for predicting the failure of wood materials when exposed to compression and shearing forces.

We then crushed roundwood to determine if round sections behaved similar to cubic sections. Round samples from dry lodgepole pine and fresh Douglas fir failed into a series of triangular segments. It appeared that the samples either split in the center or at the edges of flat as the compression area grew to be sufficient that the energy transmitted into the sample was sufficient to create Poisson's forces that split the wood. It was clear that as wedges were formed, they remained intact and were driven down through the round log piece to spread the log into an oval shape.

Figure 3a, b. The Douglas fir and lodgepole pine rounds failed into similar triangular pieces. The black lines were drawn vertical along the centerline prior to compression.

Failure in round sections appears to begin with radial cracks that propagate due to internal Poisson's stresses. We can easily explain the failure pattern by noting the contact area for round sections is not constant under load. The contact area and width of compression increases with distance compressed. In a somewhat complex way to be modeled later in this paper, once the contact area is sufficient to generate Poisson's forces that exceed the lower tensile strength perpendicular to grain, the log cracks longitudinally.

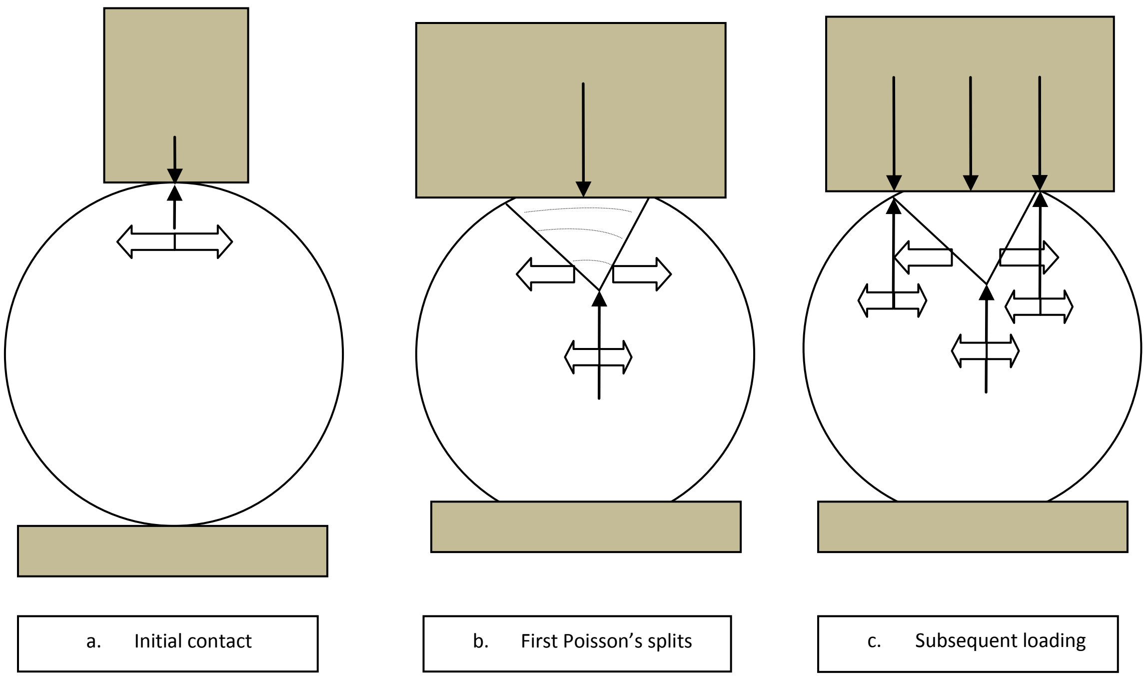


Figure 4. Depiction of the stages of failure for a round wood pole section subjected to compression.

The figure above shows the progression of loading and failure as round wood sections are compressed. We can step through the sequence of events as follows.

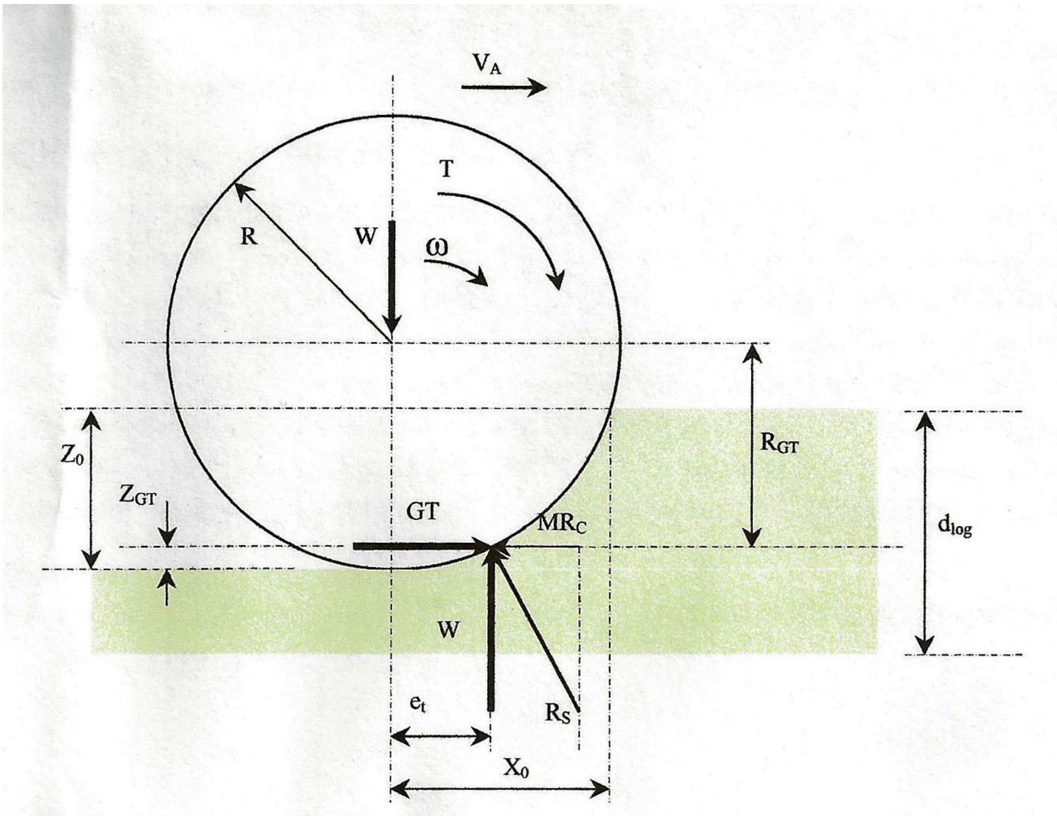
1. As the round section is compressed, the contact area initially is zero, resulting in infinite compression pressure which creates flat spots on the top and bottom of the log.
2. As the platen continues to move down, the force increases with pure compression failure and elastic compression of the log until the Poisson's force - about 40% of the downward force – triggers tensile failure tangential to the rings. The initiation of Poisson's force related failure is readily apparent in video recordings of experiments. The result is that a wedge of wood is created having a flat top and clear failure planes on the sides continuing to the center of the log.
3. The intact wedge of wood now pushes down through the log triggering rapid spreading of the log into an oval shape.
4. Continued pressure will cause flat sections on both sides of the central wedge and subsequently create new wedges outside of the center wedge as those compression areas build sufficient Poisson's force to trigger new wedges.

Figure 5. Flat plate compression of round log section vividly demonstrates the progressive failure and wedge formation due to internal Poisson's forces.

We believe that the same modes of failure apply to crushing round logs with rollers. As discussed earlier, there have been several development efforts that successfully applied roller crushers to either break roundwood into smaller sections, or in the case of scrimwood to convert a log into a fibrous mat. Unfortunately, there is no evidence in the technical nor patent literature that suggests earlier developers had either a solid understanding of exactly how their devices worked, nor had constructed mathematical models of their processes. In the next sections we will begin to develop a model for crushing round logs with smooth rollers, and then add the complexity of grooved rollers which serve to impart tangential tension forces into the log as the ribs press down. Note that the tangential tension strength is the weakest mode of failure for wood and thus may explain the observed mode of failure.

Modeling Log Crushing with Rigid Cylindrical Rollers

The calculation of energy consumption for crushing logs with round rolls is essentially the same as calculating the rolling resistance of tires on soil – except to be analogous to this case the tire would have to be running along the top of the planting ridges rather than in the furrow. The log absorbs energy as it is crushed by the rolling wheel. The forces can be resolved into a vertical crushing force (W) and a "traction" force (GT).



V_A	= forward velocity (m/s)
T	= torque (N-m)
ω	= angular velocity of the wheel (s^{-1})
W	= compression force (N)
R	= radius of compression wheel (m)
Z_0	= distance wood is compressed (m)
R_{GT}	= distance from center of wheel to line of rolling resistance in Z direction (m)
GT	= rolling resistance force (N)
R_S	= resistance vector resulting from compression force and rolling resistance (N)
M_{RC}	= GT (N)
d_{log}	= diameter of log being crushed (m)
e_t	= distance from centerline to line of rolling resistance in X direction (m)
X_0	= distance from centerline of wheel to point of initiation of compression (m)

Figure 6. Schematic of parameters for calculating compression of round logs by rolling wheels.

Approach:

We will begin by developing a method to estimate the compression force (W) as a function of the compression distance (Z_0). In the case of compression rollers having a fixed gap (probably with a spring or trip release for overloads), the distance Z_0 is a function of the log diameter and the gap set between the rollers. If the log is fairly homogeneous, Z_0 will be equal to one-half of the difference between log diameter and roller gap since half of the displacement will occur from the bottom of the log and half will be from the top. The contact area (planar area) is a half of an ellipse if the roller is moving along the log. The width of the ellipse is a function of the compression distance acting on the round log. The length of the ellipse is a function of the compression distance acting on the diameter of the roller. In the special case where the log and roller are of equal diameters, the contact area will be a half of a circle since both the width and length of the ellipse will be equal.

Until the log splits due to Poisson's forces, we may assume that the primary mechanism of compression is due to forces perpendicular to grain that create "flat spots" on the log centered on the axis of the compression rollers. Assuming that the log remains essentially round, the width of the flat spot is a chord across a circle where the diameter of the circle is the diameter of the log and the distance to the chord is the radius minus the compression distance Z_0.

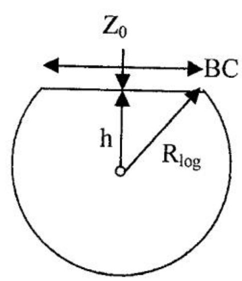


Figure 7. Schematic showing parameters for calculation of chord length (BC) for roundwood under compression from above.

The solution of the chord length as a function of Z_0 is a straightforward geometry problem.

$$\text{Chord length (BC)} = 2*\sqrt{(R_{log}^2+h^2)} \text{ where } h = R_{log}-Z_0 \qquad \text{Equation 2}$$

Where:

R_{log} = radius of log being crushed (1/2 of d_{log})

h = distance from center of log to lower edge of crushing roller

Z_0 = distance crushed from surface of log

Now that we know the chord distance (BC) we can turn to the calculation of the length (AD) of the ellipse that describes the contact between the roller and log.

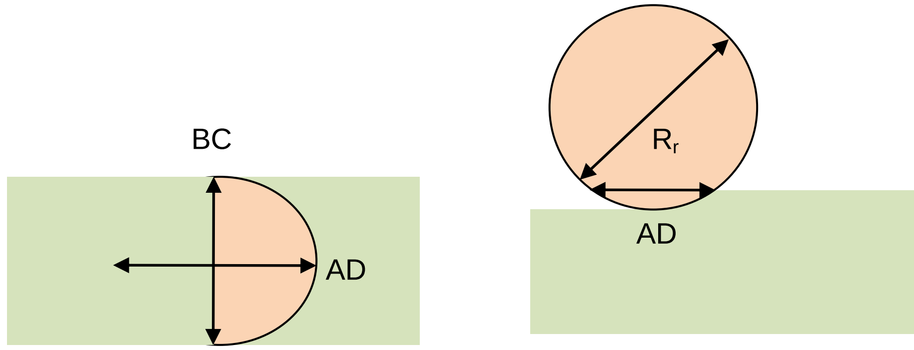


Figure 8. Schematic of plan view of elliptical contact area between large diameter roller and round wood log showing the length of the contact ellipse (AD) and width (BC).

The width of the ellipse (BC) was calculated from the round wood as above. The length of the ellipse (AD) is also a chord length for the same Z_0 compression distance applied to the diameter of the roller. The only change to the above equation is to substitute the radius (R_r) of the roller for the radius of the log.

Now that we have described the ellipse by its length and width, we can again apply plane geometry to calculate the area (A) of one-half of the ellipse.

$$A = \pi *(AD/2)*(BC/2)/2 \qquad \text{Equation 3}$$

As an example, consider the following log and roller combination:

Diameter of roller = 400 mm

Diameter of log = 200 mm

Compression distance Z_0 = 25 mm

The chord length (BC) for the width of the contact area on the log is calculated to be 132 mm.

The chord length (AD) for twice the length of the contact area on the roller is calculated to be 194 mm.

From these values, the contact area is calculated to be 10,060 mm^2 or 0.010 m^2.

We also need to determine the location of the force center, which is coincident with the centroid of the area of the ½ ellipse. The centroid is found by the moment of inertia divided by the area. We've already found the area to be A = π *(AD/2)*(BC/2)/2 or A = AB*BC*π/4. To simplify the moment calculation, a ¼ ellipse is evaluated for moment in the x direction.

Cartesian coordinates of an ellipse

If we assume that an ellipse has a length (a) and width (b) and that (x) designates length along the long axis and (y) designates width along the width axis, then:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow f^2(x) = b^2 - \frac{x^2 b^2}{a^2} \Rightarrow f(x) = \frac{b}{a}\sqrt{a^2 - x^2}$$ Equation 4

$$M_y = \int_0^a x f(x) dx$$ Equation 5

$$A = \frac{ab\pi}{4}$$ Equation 6

$$\bar{x} = \frac{My}{A} = \frac{4}{ab\pi}\int_0^a x \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$ Equation 7

Use $u = a^2 - x^2$ and $du = -2xdx \Rightarrow \frac{-1}{2} du = xdx$ as substitution

And limits of integration change such that when u = 0, x= a^2, and when u = a, x=0.

Now

$$\bar{x} = \frac{4}{a^2\pi}\int_{a^2}^0 \sqrt{u}\, \frac{-1}{2} du$$ Equation 8

Then $\bar{x} = 0 - \left[\frac{-2}{a^2\pi}\frac{2}{3}\left(a^2\right)^{\frac{3}{2}}\right] = \frac{4a}{3\pi}$ Equation 9

We can finally bring the resulting equation back into the nomenclature of Figure 6.

$e_{t} = 2 * X_0 / 3 * \pi$

Equation 10

Where:

e_t = distance from centerline to line of rolling resistance in X direction (m)

X_0 = distance from centerline of wheel to point of initiation of compression (m)

The multiplier on X_0 is only 2 since X_0 is equal to one-half the length of the ellipse.

Relating contact area to compression force and Poisson's forces

We can now back-calculate to estimate the area needed to generate sufficient internal Poisson's forces to begin longitudinal cracks in the log.

Continuing with our example, if the species is Douglas fir and the moisture content is high, from **Table 2** we would expect the compression strength perpendicular to grain to be 2,600 kPa and the tensile strength tangential to grain to be 2,100 kPa. We also know from **Table 1** that the Poisson's ratio is about 0.4.

We are going to make a heroic assumption at this point that Poisson's force-related cracks begin at the end of the log as compression is initiated and propagate down the log as driven by energy generated by the roller acting to compress the log.

Model Validation

A spreadsheet was created in Microsoft Excel® to calculate crushing forces and energy following our model. Our first case is shown below in which the roller was assumed to be 300 mm diameter and was used to crush a 100mm diameter log typical of forest thinnings or residuals.

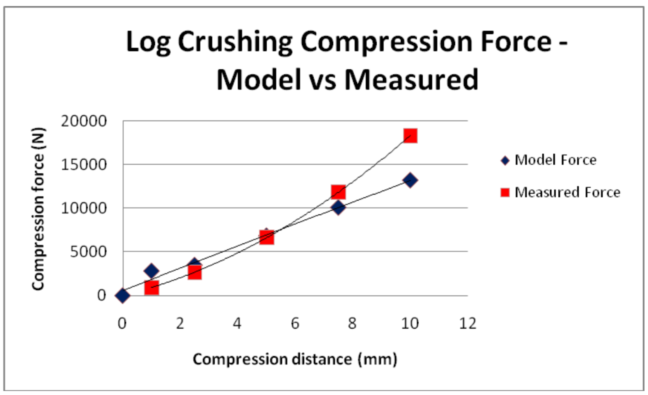


Figure 9. Graph of the calculated compression force (N) versus measured force for a 300 mm diameter roller acting on a 100 mm diameter round log.

It is evident from the graphical plot that the model appears to be more linear than reality suggests. We suspect that an additional component will be needed to account for other modes of compressive stress accumulation within the wood. However, we were quite pleased that our first-effort results were within the same order of magnitude when comparing the model to a set of laboratory measurements.

Conclusion

We were able to develop a mathematical model for log crushing forces that had a reasonable fit with measured values. This kind of model has been called for by several programs over the past 40 years, but this appears to be the first publication of a prototype model. In agreement with our sponsors at the US Department of Energy, we encourage others to improve the model and add components that improve its predictive power.

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